Optimization

Dynamic programming is very useful in solving optimization problems.

1.Find the maximum

2.Find the minimum

3.Find the shortest

4.Find the longest

5.Maximum profit

6.Minimum cost

7.Optimum

8.Count number of ways

9.Minimum number of moves

10.Fewest

11.Find the Smallest

12.Find the largest

A given problem has **Optimal Substructure Property** if optimal solution of the given problem can be obtained using optimal solutions of its subproblems.

Coin Change

You are given coins of different denominations and a total amount of money amount. Write a function to compute the fewest number of coins that you need to make up that amount.

coins = [1, 2, 5], amount = 11

Output : 3 , 5+5+1 = 11

2. There are a row of n houses, each house can be painted with one of the three colors: red, blue or green. The cost of painting each house with a certain color is different. You have to paint all the houses such that no two adjacent houses have the same color.

Multi stage decision process

- Multistage decision problems are those in which decisions are made in multiple stages. In each stage we have multiple decisions, our goal is to consider every decision and choose the one which gives the most optimal result.
- Also called sequential decision problems.

Multistage optimization

- The process of solving multistage decision problems is called multistage optimization.
- A sequence of decisions will be called policy or a program,
- A policy which is most advantageous according to some preassigned criterion will be called optimal policy.

States

A state is defined by set of parameters whose values reflect the information required to assess the consequences that the current decision has upon future actions. Based on which decision we take the state changes and it becomes the input to the next stage.

0-1 Knapsack problem

Knapsack
capacity W = 20 lbs

	Item	Weig ht	Value
0		3	4
1		7	14
2		10	10
3		6	5

Knapsack

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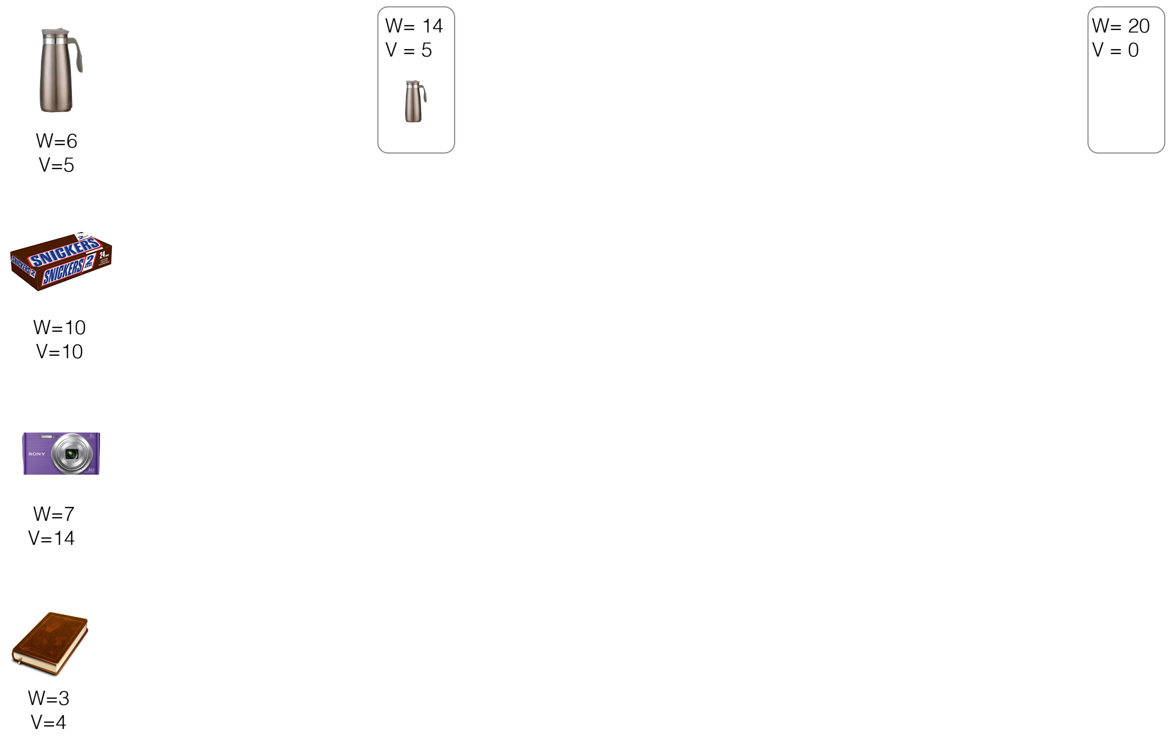
V=5

W=10
V=10

W=7
V=14

W=3
V=4

Knapsack



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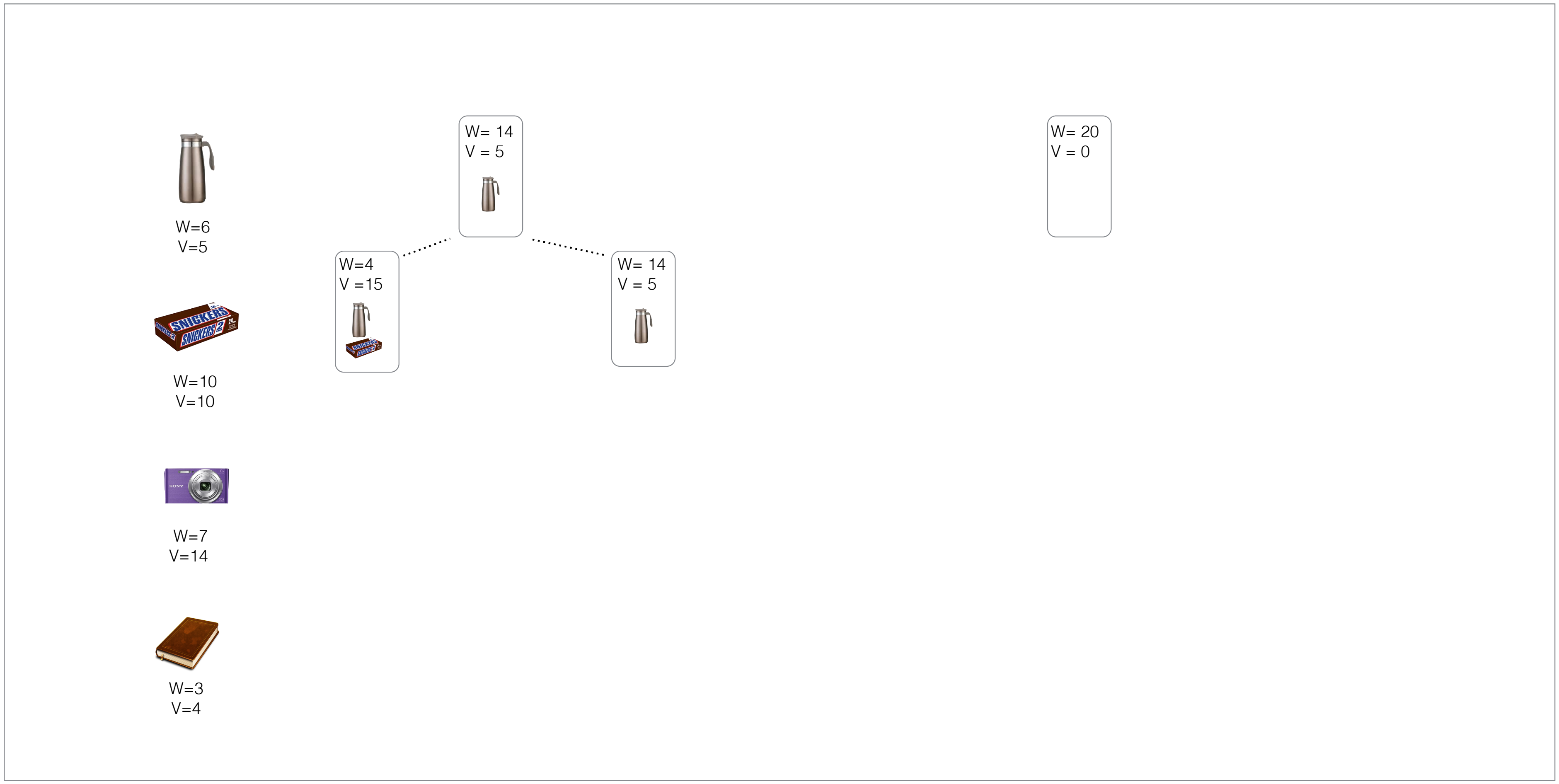

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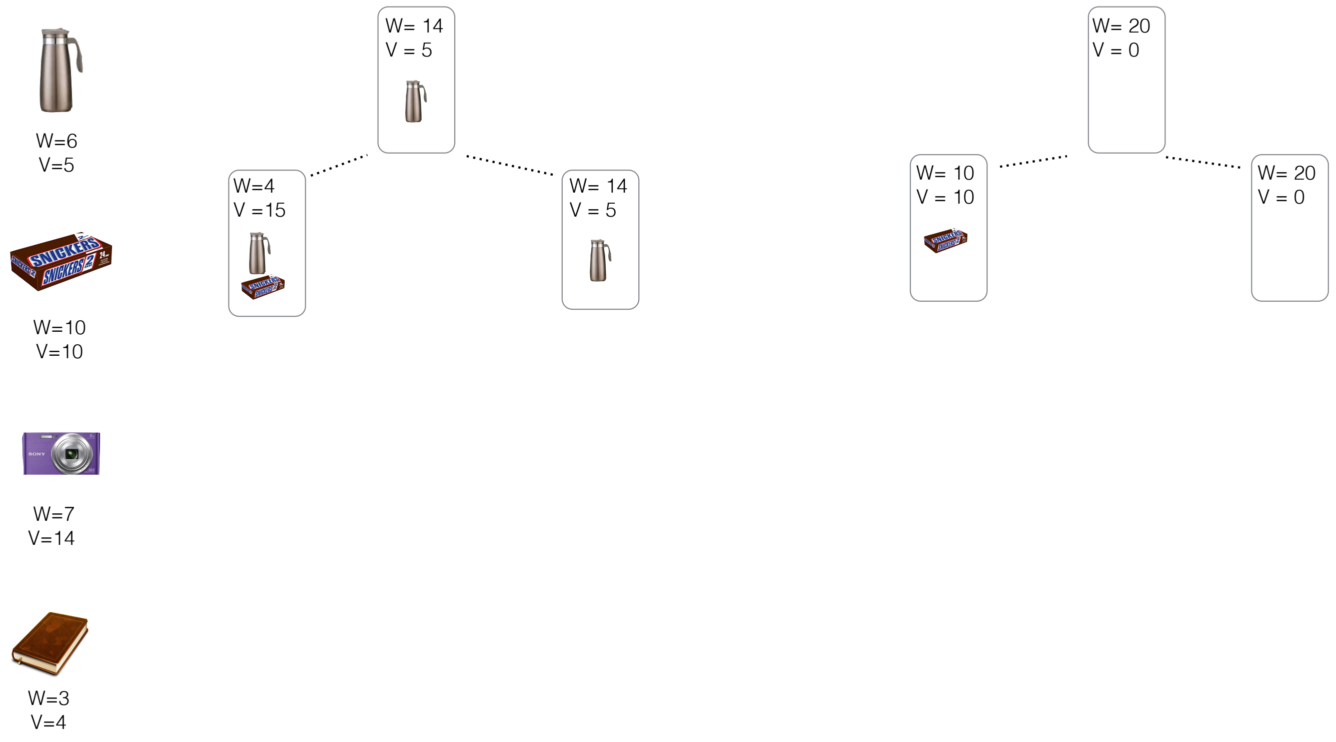
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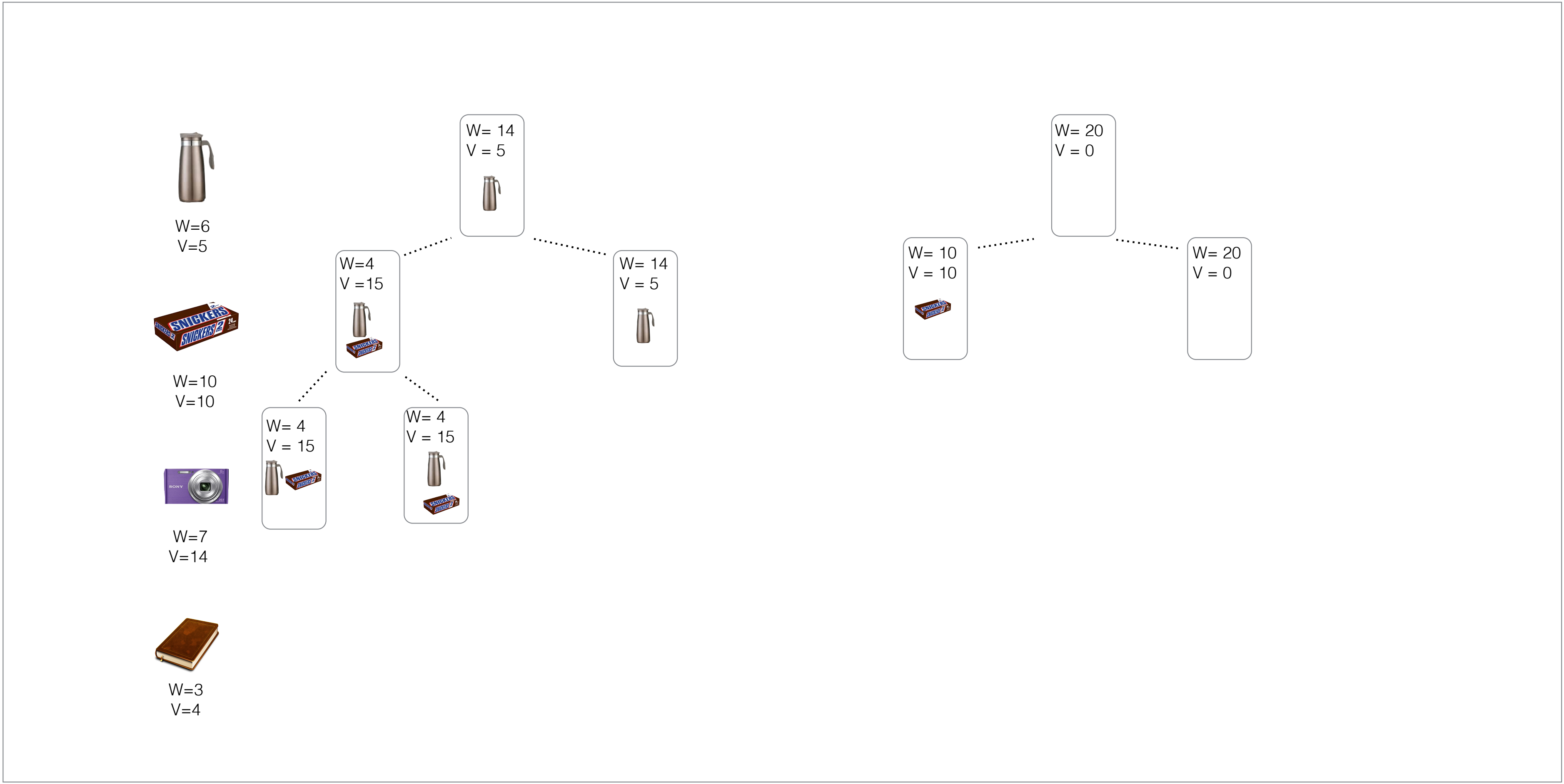
V = 0

Knapsack



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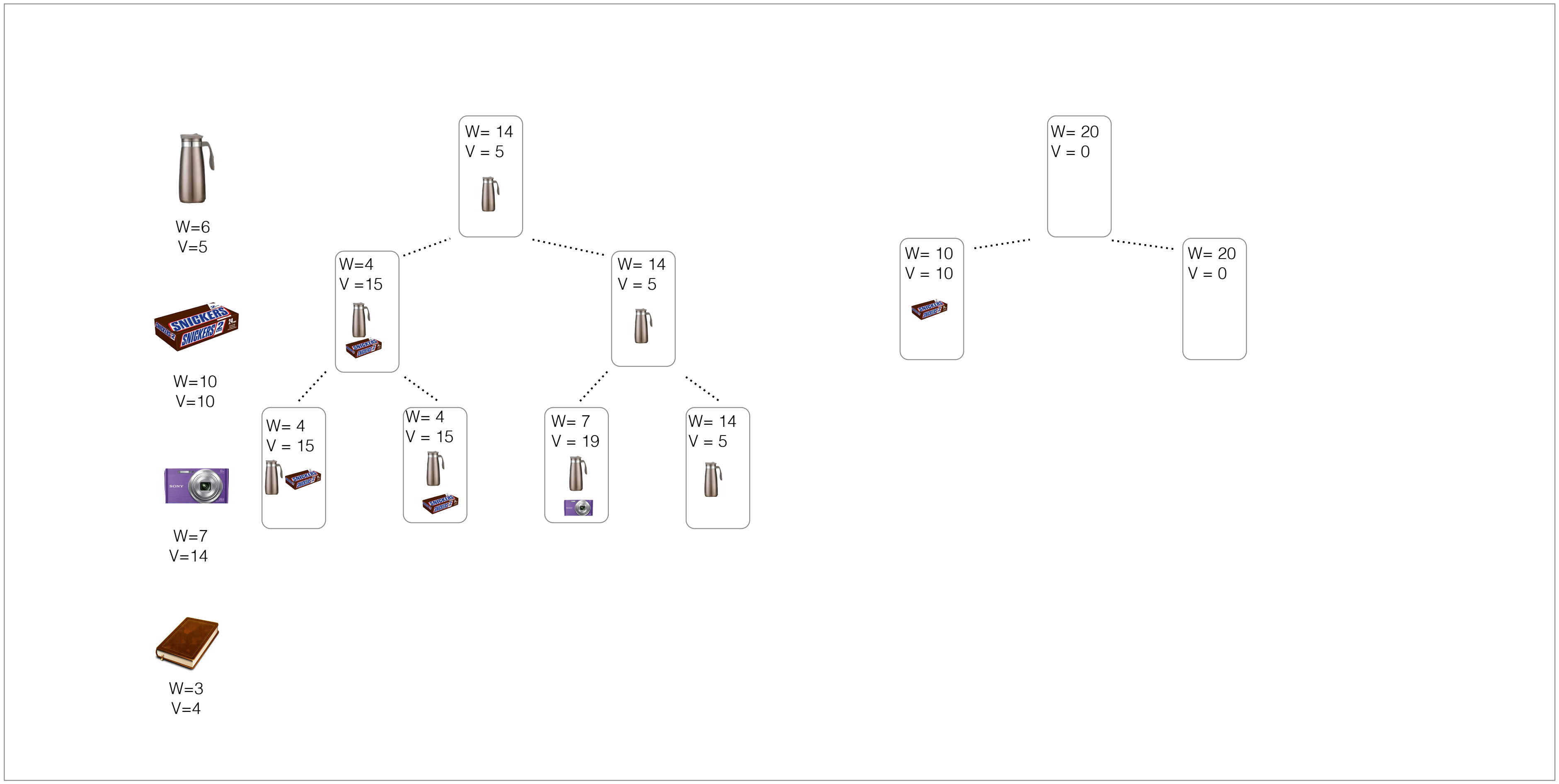

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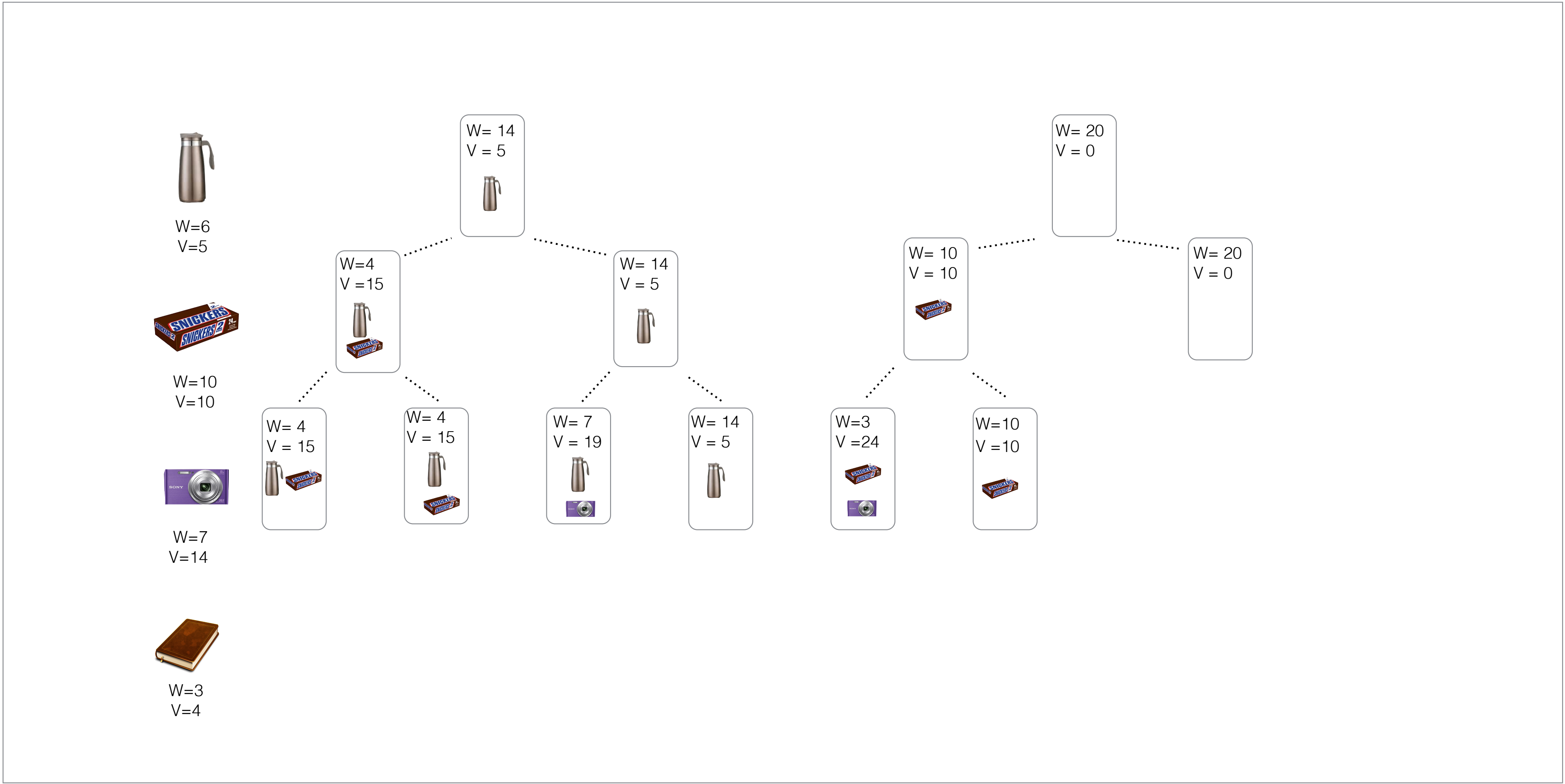
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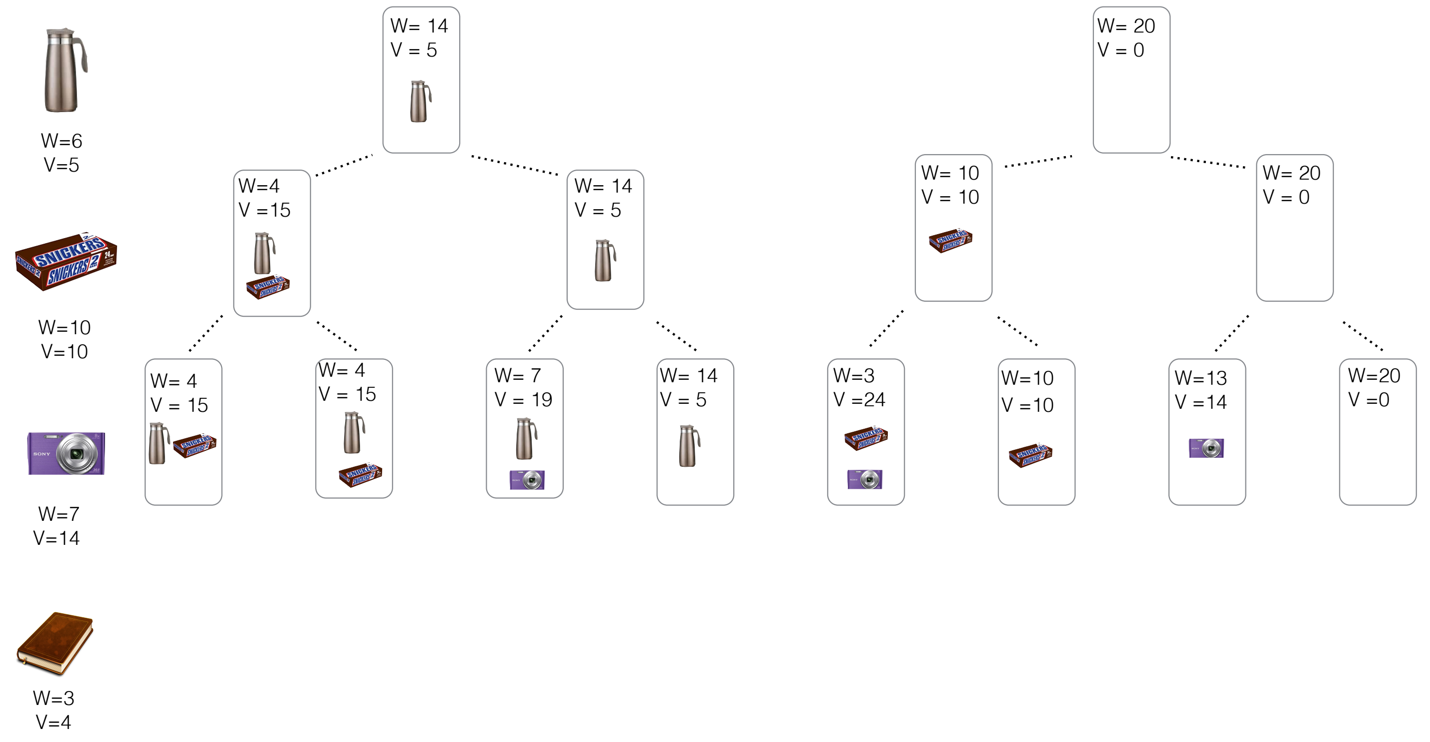

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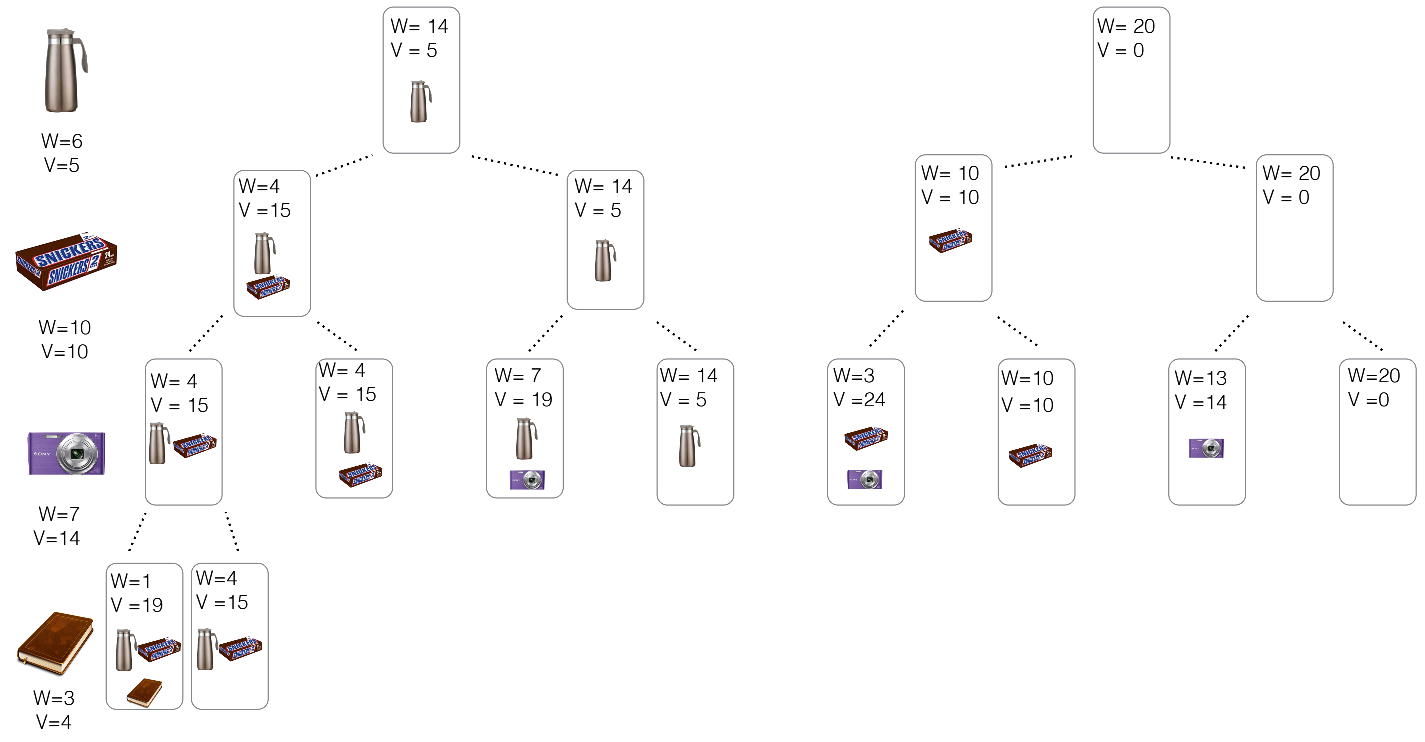
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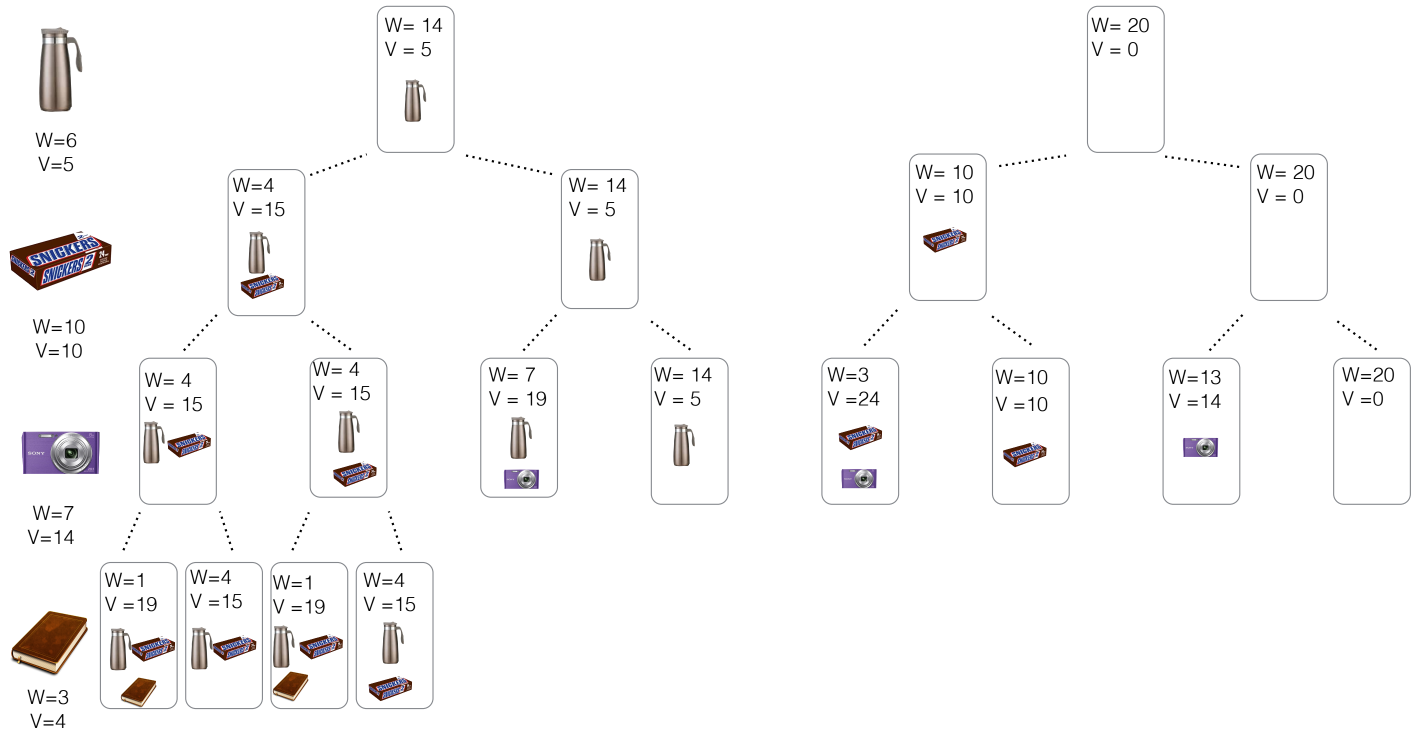
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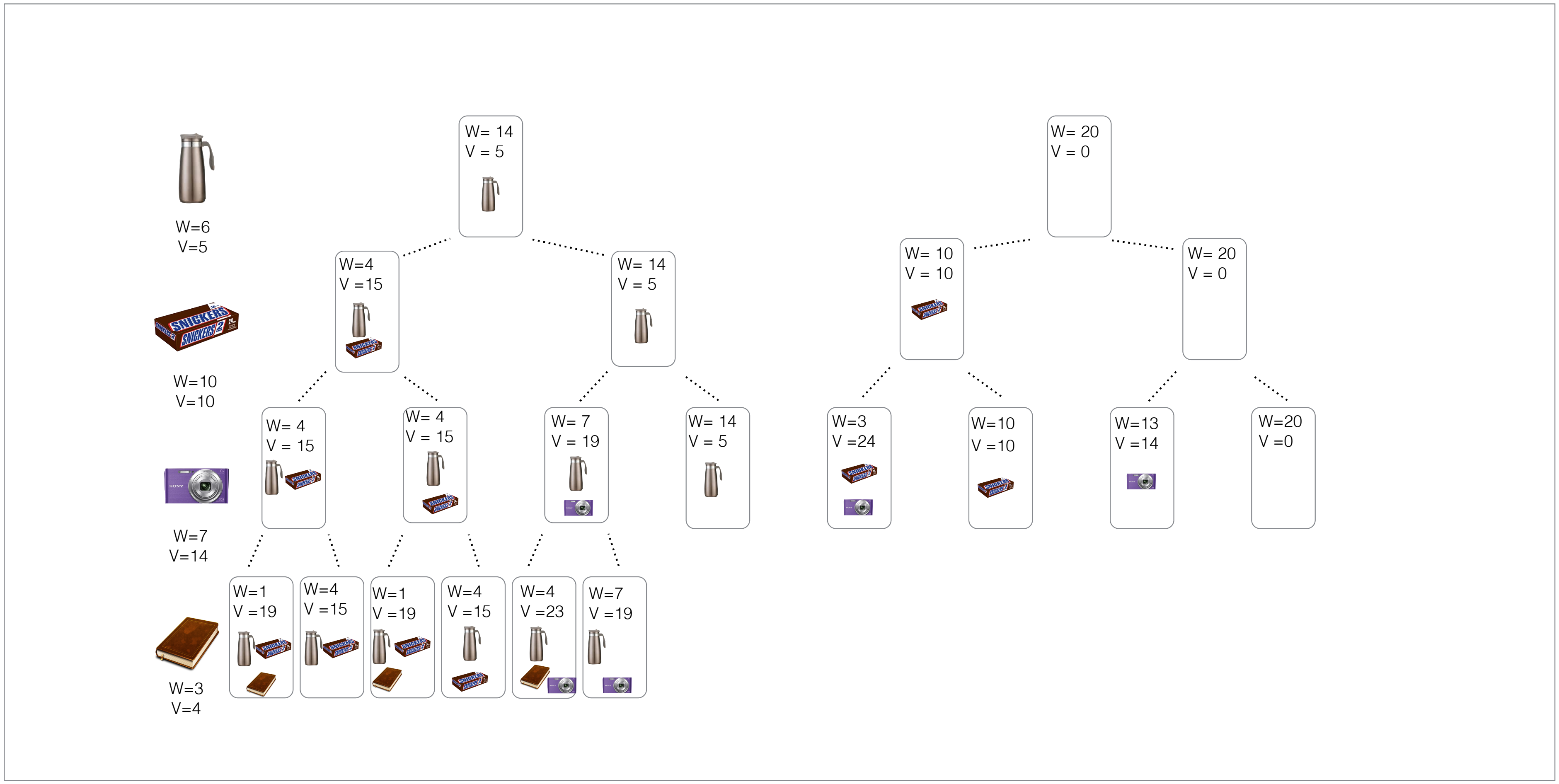
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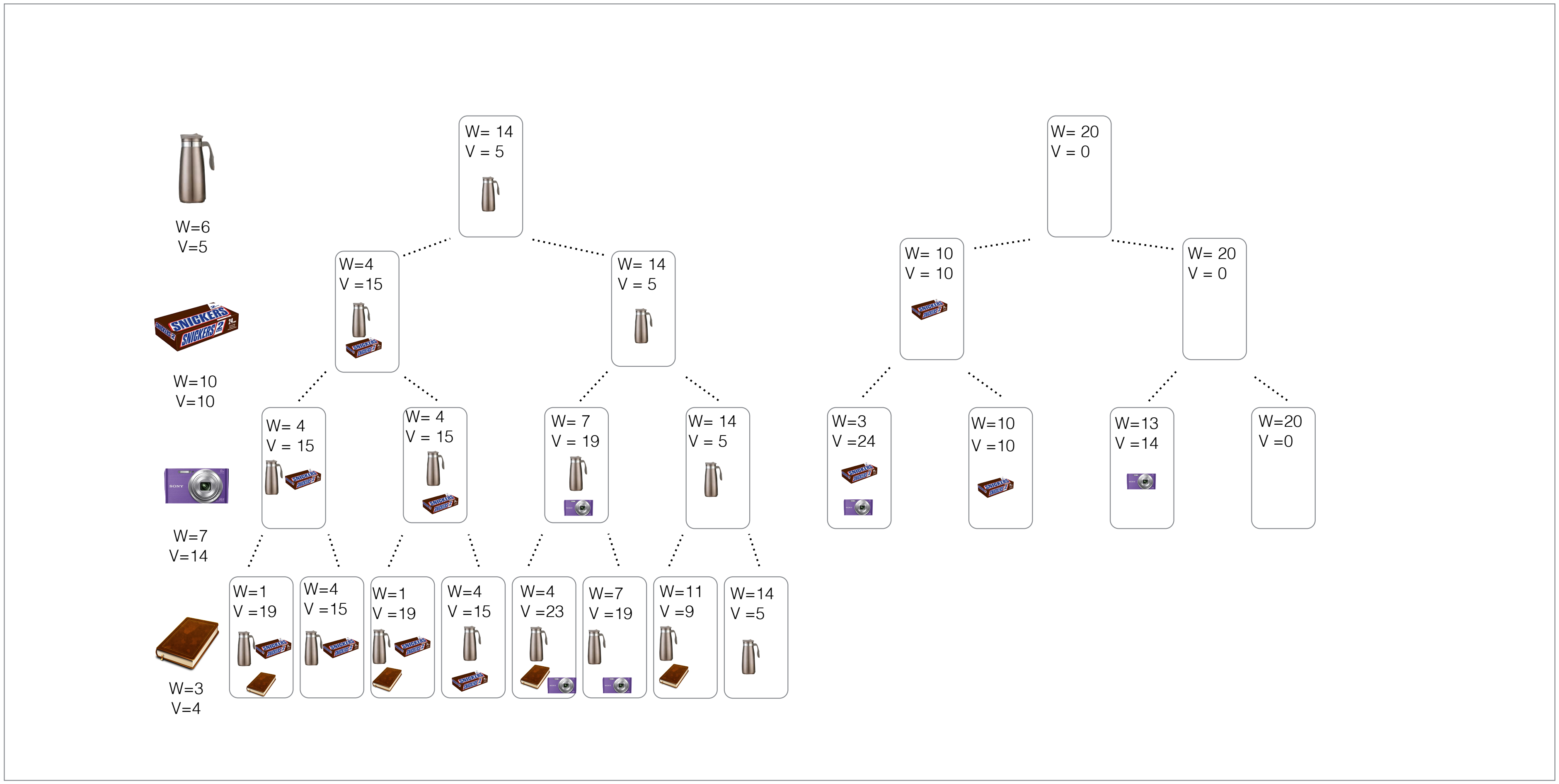

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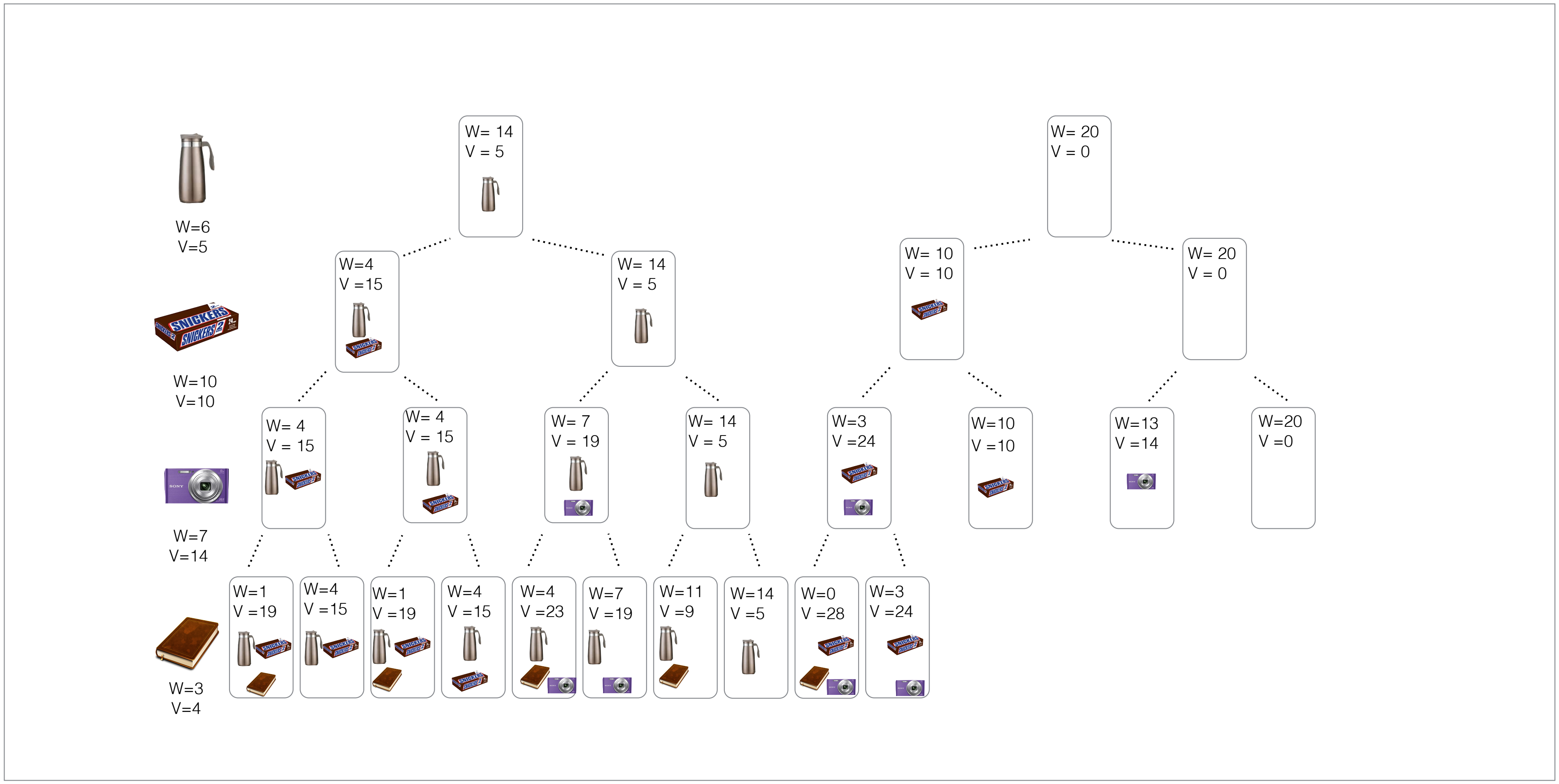
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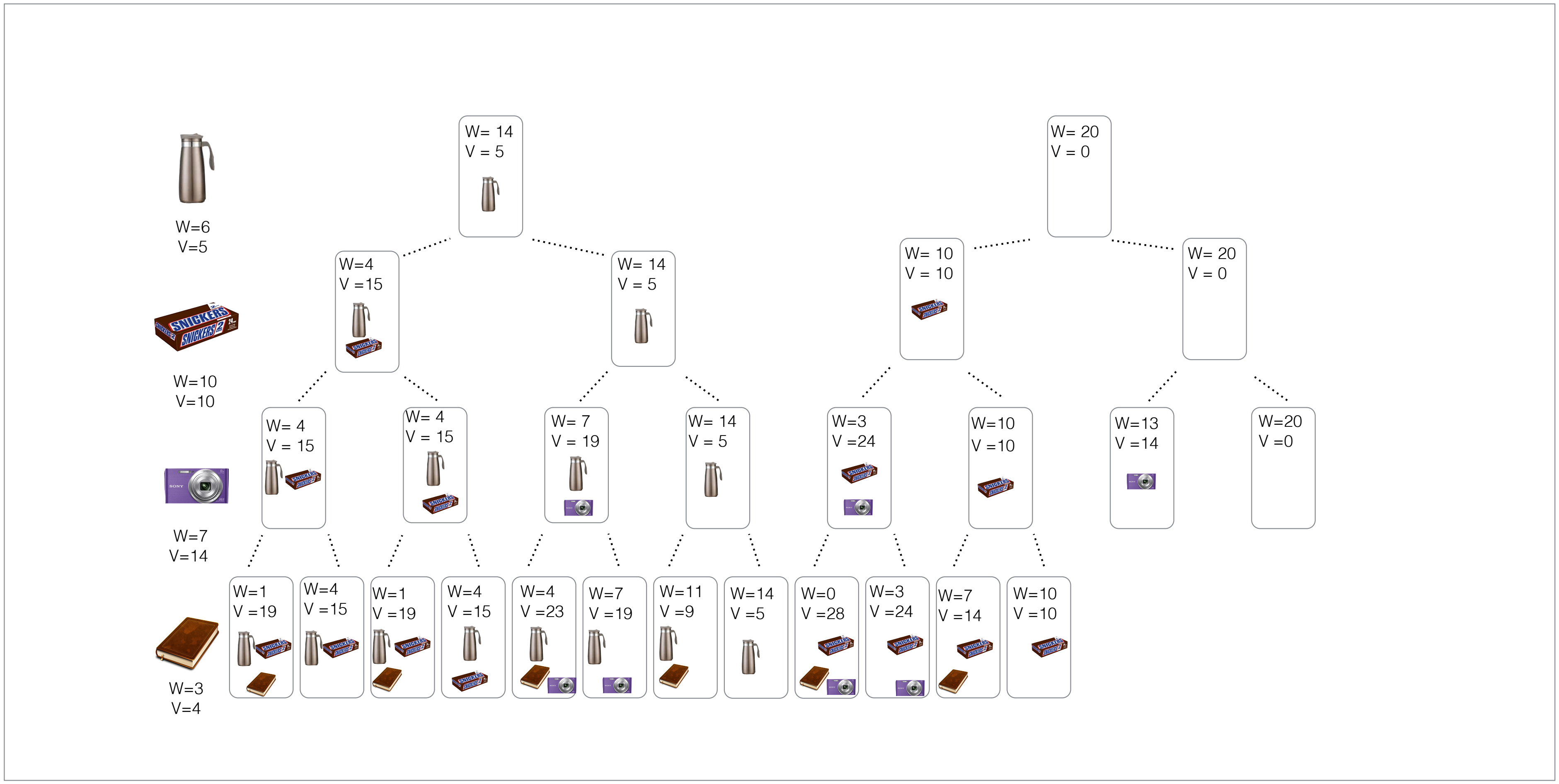

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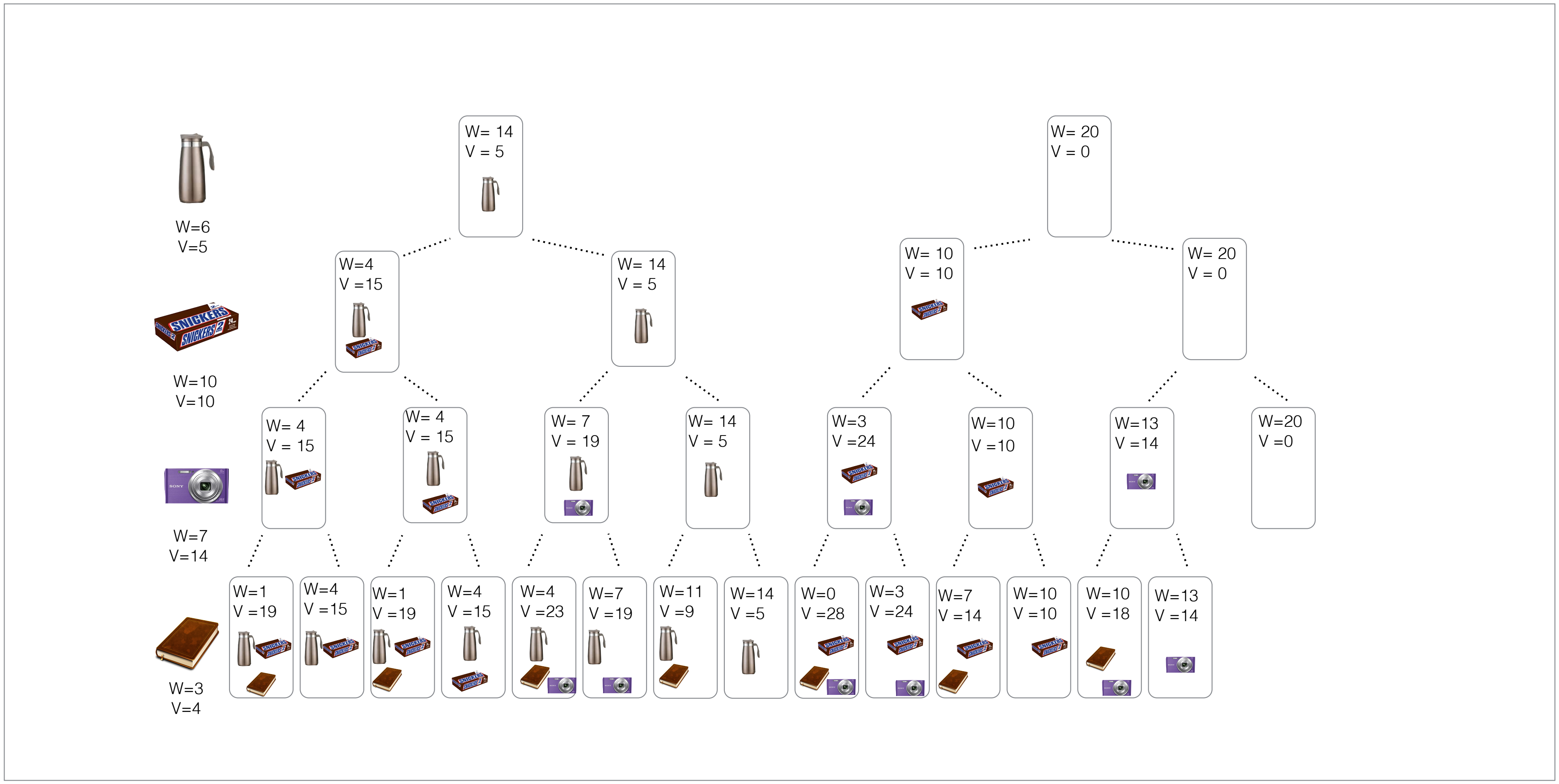
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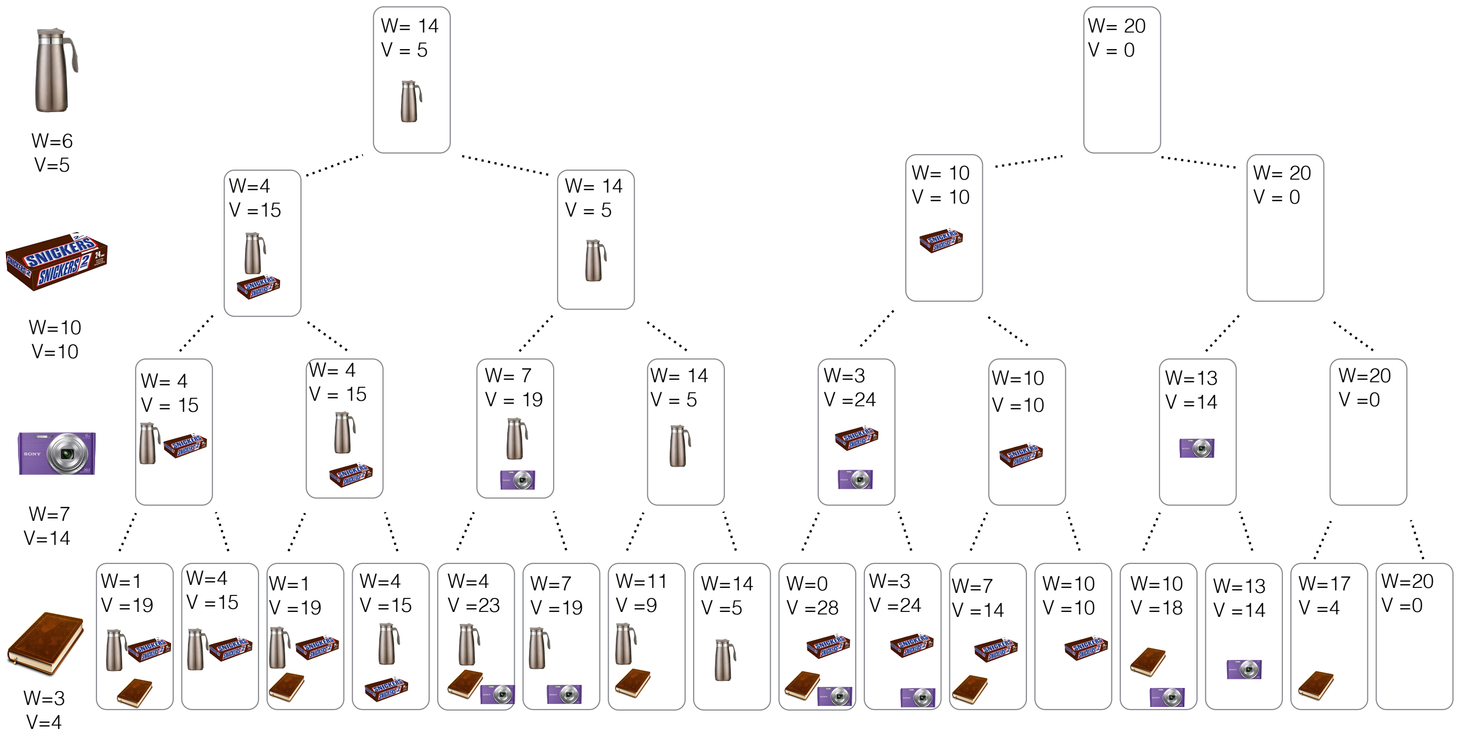
Knapsack



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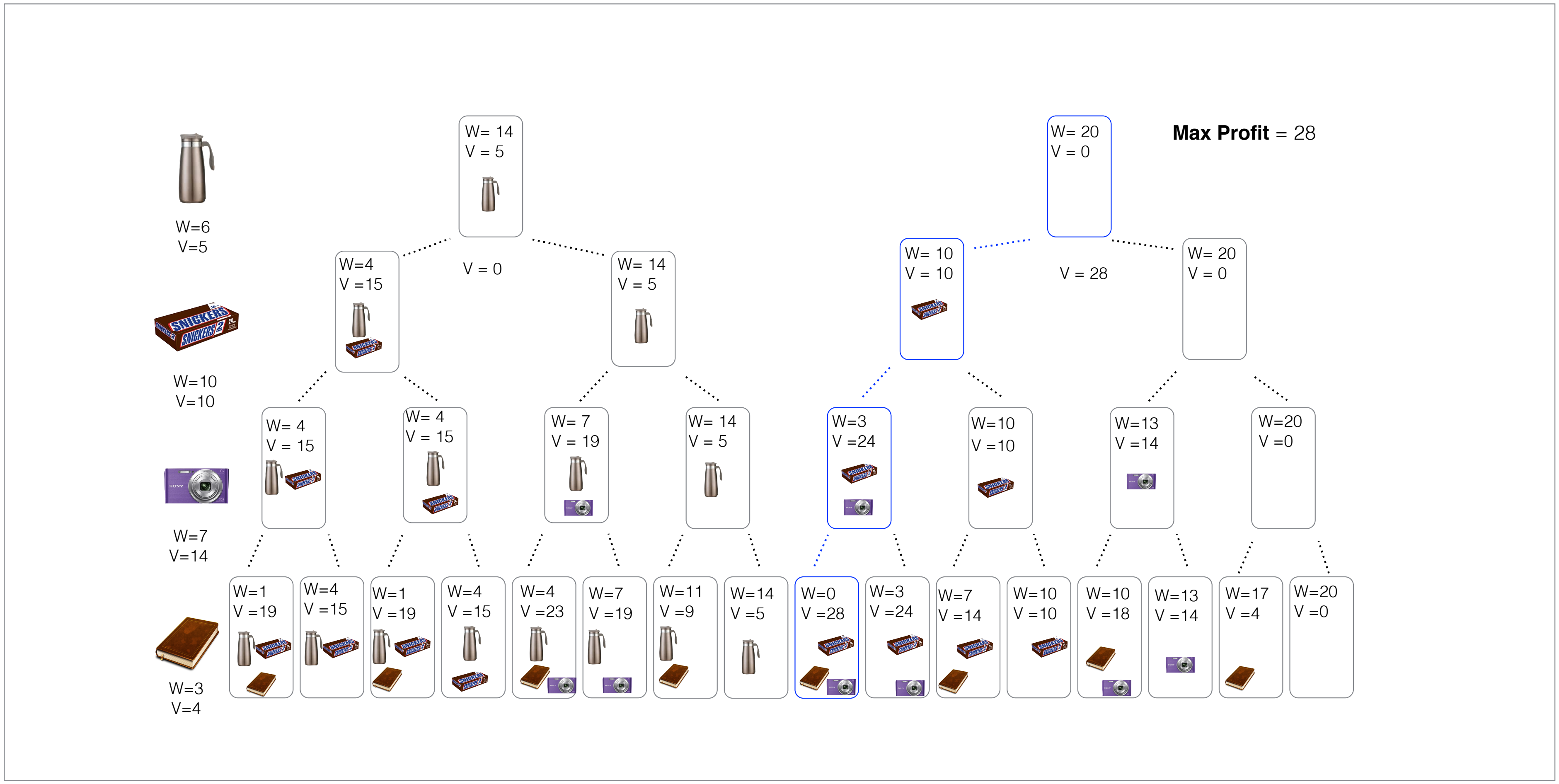
Knapsack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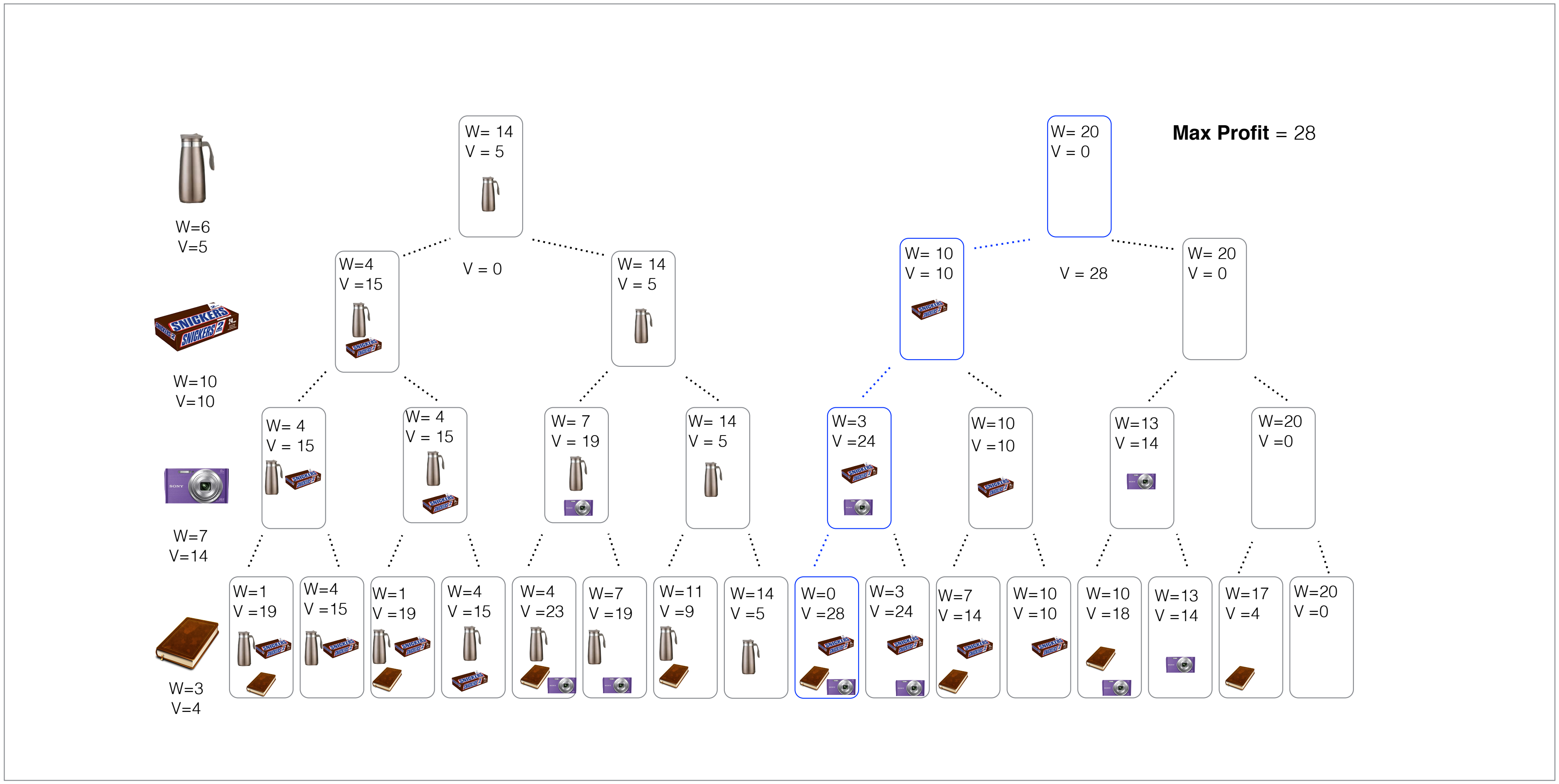
Principal of optimality

An optimal policy (or a set of decisions) has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision.”

Knapsack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V =0

Recursive Optimization

- Because of this multistage decision nature of the optimization process we can come up with *recursive optimization* procedure, which builds to a solution to the overall N -stage problem by first solving one-stage problem at a time sequentially until the overall optimum has been found.

- Each recursive equation represents a stage at which a decision is required.

- We use the same recursive function at every stage.